

DETERMINING THE STRUCTURAL BREAK IN SOME ECONOMIC VARIABLES OF PAKISTAN

By

Khalil Ahmad

Structural Breaks Detection

Bai and Perron established a general methodology for estimating breakpoints and their associated confidence intervals in OLS regression employing dynamic programming. In that way, it is possible to find m breakpoints that minimize the residual sum of square (RSS) associated to a model with $m+1$ segments given some minimal segment size of $h*n$ observations. The h bandwidth parameter is chosen by the user typically equal to 0.1 or 0.15. Since the number of breakpoints m is not known in advance, it is necessary to compute the optimal breakpoints for $m = 0, 1, \dots$ breaks and choose the model that minimizes some information criterion such as BIC.

Structural Breaks Analysis

In the following, we determine the Exchange Rate time series structural changes dates, if any. Such analysis is named as “dating structural changes (breaks)”. Specifically, we are looking for:

- * Level Structural Breaks
- * Trend Structural Breaks
- * Polynomial Fit Structural Breaks
- * Auto-Regressive Model Structural Breaks

3.1 Research Design:

Appropriate quantitative techniques will be applied for the finding of this study. The method will be compare to find the adequate and appropriate technique to forecast the values of the water outflow data.

3.2 Data Collection and analysis

Secondary time series data of water outflow is used from 2015 to 2019 recorded on daily basis. Appropriate and adequate statistical method have been applied using R, a statistical software.

Determining the Structural Breaks in Exchange Rate of Pakistan

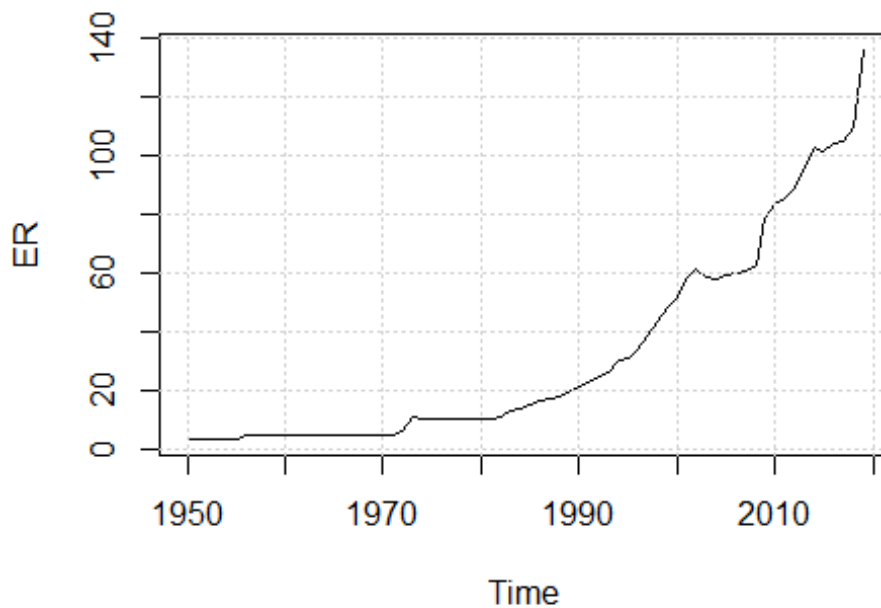
In time series analysis, structural changes represent shocks impacting the evolution with time of the data generating process. That is relevant because one of the key assumptions of the Box-Jenkins methodology is that the structure of the data generating process does not change over time. How can structural changes be identified?

Basic Data Exploration

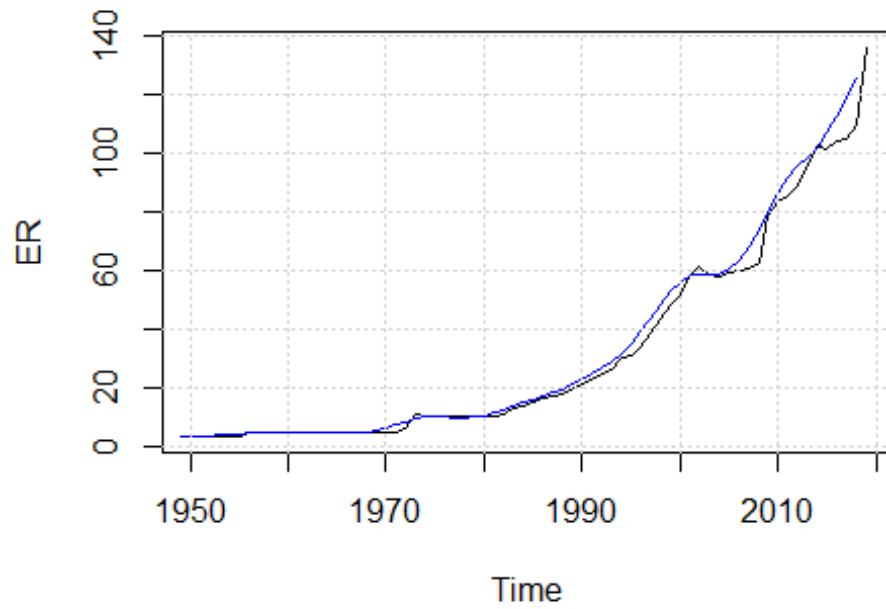
Summary of the Data Series

##	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
##	3.309	4.770	14.330	32.229	58.222	136.090

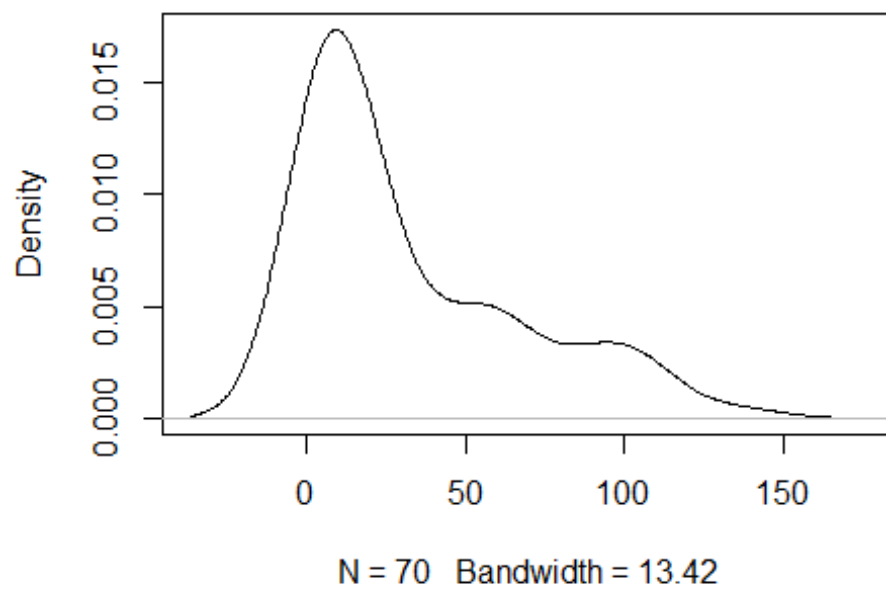
Time Series Plot of Exchange Rate



Time Series Plot of Exchange Rate with Smooth Fit



Density Plot of Exchange Rate



Structural Changes Detection

Bai and Perron established a general methodology for estimating breakpoints and their associated confidence intervals in OLS regression employing dynamic programming. In that way, it is possible to find m breakpoints that minimize the residual sum of square (RSS) associated to a model with $m+1$ segments given some minimal segment size of $h \cdot n$ observations. The h bandwidth parameter is chosen by the user typically equal to 0.1 or 0.15. Since the number of breakpoints m is not known in advance, it is necessary to compute the optimal breakpoints for $m = 0, 1, \dots$ breaks and choose the model that minimizes some information criterion such as BIC.

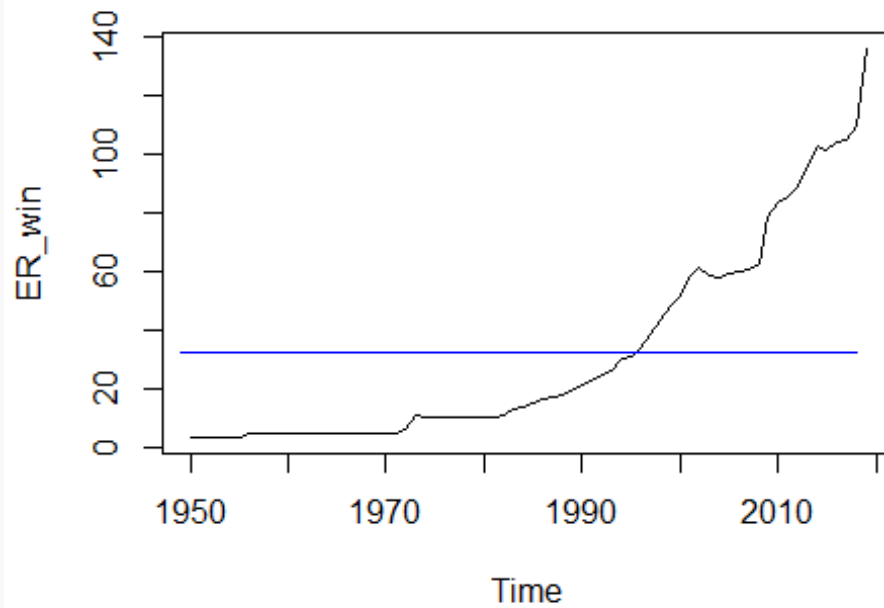
Structural Changes Analysis

In the following, we determine the Exchange Rate time series structural changes dates, if any. Such analysis is named as “dating structural changes (breaks)”. Specifically, we are looking for:

- * Level Structural Changes
- * Trend Structural Changes
- * Polynomial Fit Structural Changes
- * Auto-Regressive Model Structural Changes

Summary of Level Fit

```
Residuals:
##      Min       1Q   Median       3Q      Max
## -28.92 -27.46 -17.90  25.99 103.86
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   32.229      4.167    7.734 6.12e-11 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 34.86 on 69 degrees of freedom
```



Summary regarding Breaks

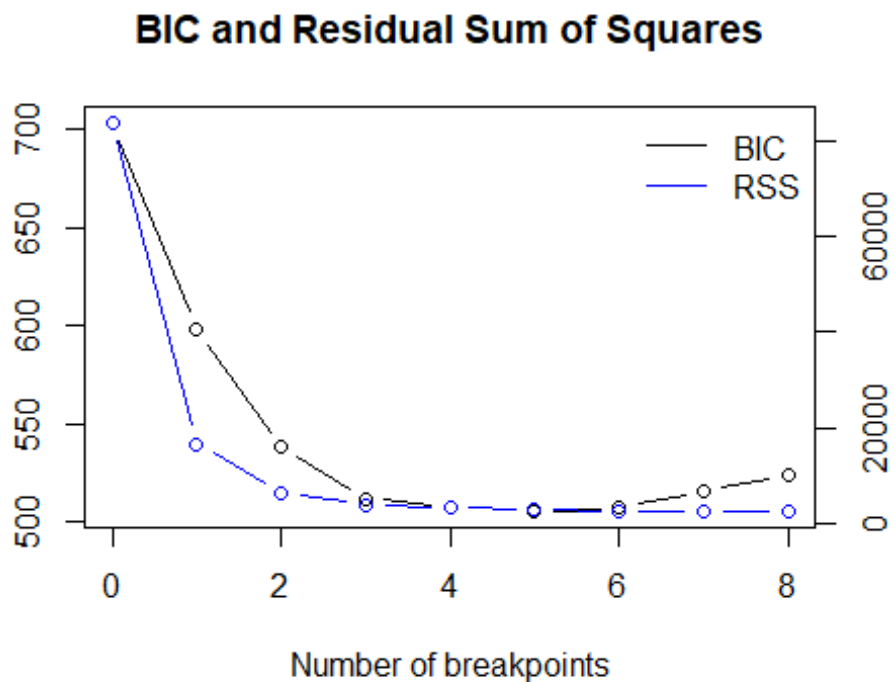
```
## ##
##   Optimal (m+1)-segment partition:
##
## Call:
## breakpoints.formula(formula = ER_win ~ 1, h = 0.1)
##
## Breakpoints at observation number:
##
## m = 1           49
## m = 2           46 59
## m = 3           36 48 60
## m = 4           33 42 49 60
## m = 5           33 42 49 56 63
## m = 6           23 35 42 49 56 63
## m = 7    7     23 35 42 49 56 63
## m = 8    7 16 23 35 42 49 56 63
##
## Corresponding to breakdates:
##
## m = 1           1998
## m = 2           1995 2008
## m = 3           1985     1997     2009
## m = 4           1982 1991 1998     2009
## m = 5           1982 1991 1998 2005 2012
## m = 6           1972 1984 1991 1998 2005 2012
## m = 7    1956     1972 1984 1991 1998 2005 2012
```

```

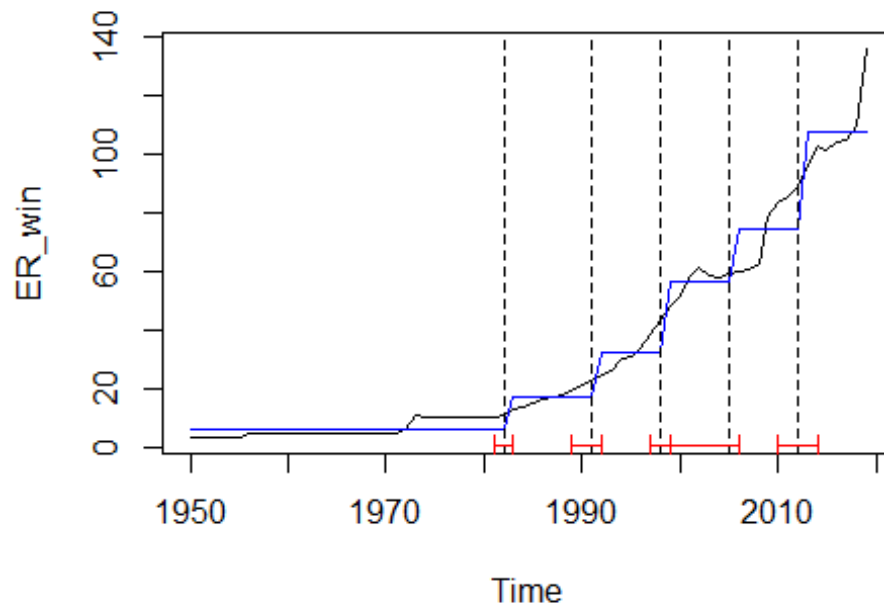
## m = 8    1956 1965 1972 1984 1991 1998 2005 2012
##
## Fit:
##
## m    0      1      2      3      4      5      6      7
## RSS 83866.8 16584.4  6205.0  3798.1  3170.5  2714.4  2466.3  2457.8
## BIC  703.3   598.4   538.1   512.2   508.1   505.7   507.5   515.7
##
## m    8
## RSS  2457.7
## BIC   524.2

```

Above the results of finding $m = 1$ to 8 breakpoints with associated dates and $\{RSS, BIC\}$ metrics. The minimum value of BIC is reached for $m = 5$. Graphically, we have the following plot



The plot of the observed and fitted time series, along with confidence intervals for the breakpoints, is given by:



The break dates are

1982 1991 1998 2005 2012

Level breaks coefficients are

```
##          (Intercept)
## 1950 - 1982      6.131882
## 1983 - 1991     17.262322
## 1992 - 1998     32.425300
## 1999 - 2005     56.498371
## 2006 - 2012     74.296414
## 2013 - 2019    107.963957
```

Trend Structural Changes

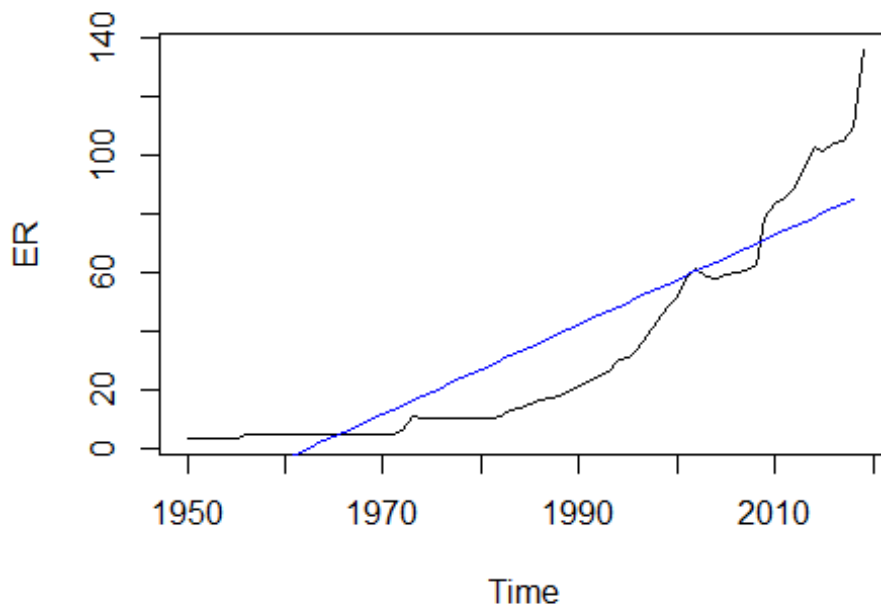
Trend structural changes can be determined as

Summary

```
## Residuals:
##      Min       1Q   Median       3Q      Max
## -19.984 -13.669  -4.128  12.687  51.078
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -22.08481    3.81755  -5.785 2.01e-07 ***
## tt           1.52995    0.09346  16.370 < 2e-16 ***
```

```
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 15.8 on 68 degrees of freedom
## Multiple R-squared:  0.7976, Adjusted R-squared:  0.7946
## F-statistic: 268 on 1 and 68 DF, p-value: < 2.2e-16
```

Both intercept and slope coefficients are reported as significant. Let us plot the time series against the fit.



```
## Breakpoints at observation number:
##
## m = 1          41
## m = 2          39  55
## m = 3          35  49  59
## m = 4          32  45  52  59
## m = 5          23  32  45  52  59
## m = 6          23  31  38  45  52  59
## m = 7          7   23  31  38  45  52  59
## m = 8          7  16  23  31  38  45  52  59
##
## Corresponding to breakdates:
##
## m = 1          1990
## m = 2          1988      2004
## m = 3          1984      1998  2008
## m = 4          1981      1994  2001  2008
```

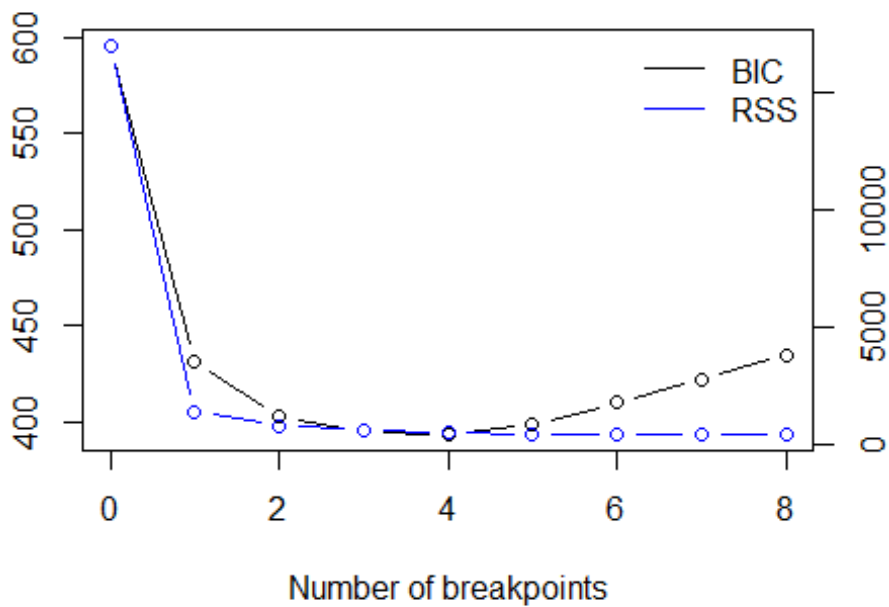


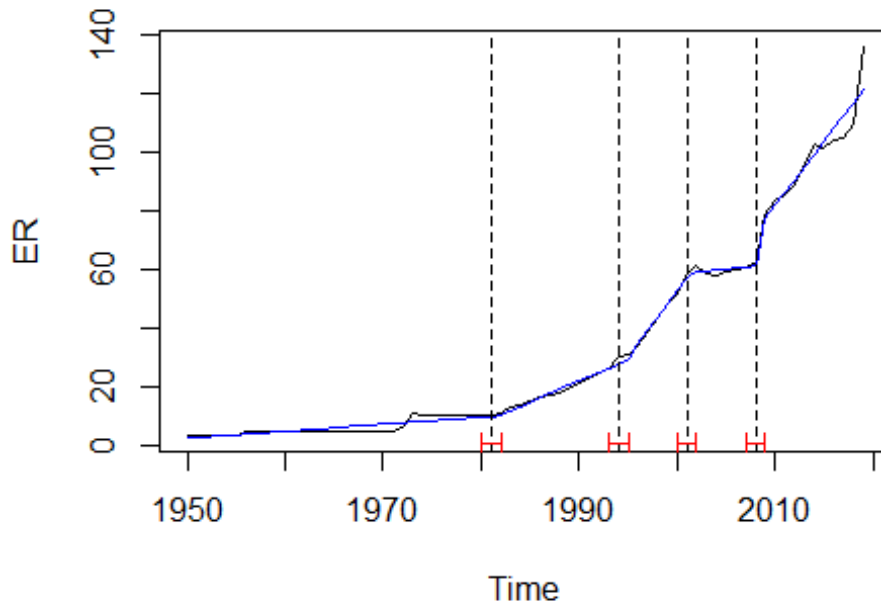
```

## m = 5          1972 1981          1994 2001 2008
## m = 6          1972 1980 1987 1994 2001 2008
## m = 7  1956    1972 1980 1987 1994 2001 2008
## m = 8  1956 1965 1972 1980 1987 1994 2001 2008
##
## Fit:
##
## m  0      1      2      3      4      5      6      7
## RSS 16973.7 1351.1  753.4  561.7  456.7  408.8  400.4  398.4
## BIC  595.8  431.4  403.2  395.4  393.7  398.7  410.0  422.3
##
## m  8
## RSS  398.1
## BIC  435.0

```

BIC and Residual Sum of Squares





Trend Break dates are

```
## [1] 1981 1994 2001 2008
```

Trend Break Coefficients are

```
##          (Intercept)          tt
## 1950 - 1981    2.017539  0.2409590
## 1982 - 1994  -37.524566  1.4491368
## 1995 - 2001 -182.566250  4.6140071
## 2002 - 2008   40.157757  0.3540571
## 2009 - 2019 -186.822459  4.4025627
```

Where tt is stands for time trend

Polynomial Fit Structural Changes

Second degree polynomial structural changes can be determined as

Coefficients:

```
##          Estimate Std. Error t value Pr(>|t|)
## (Intercept) 12.554169   1.778427   7.059 1.19e-09 ***
## tt          -1.356626   0.115593 -11.736 < 2e-16 ***
## I(tt^2)      0.040656   0.001578  25.768 < 2e-16 ***
```

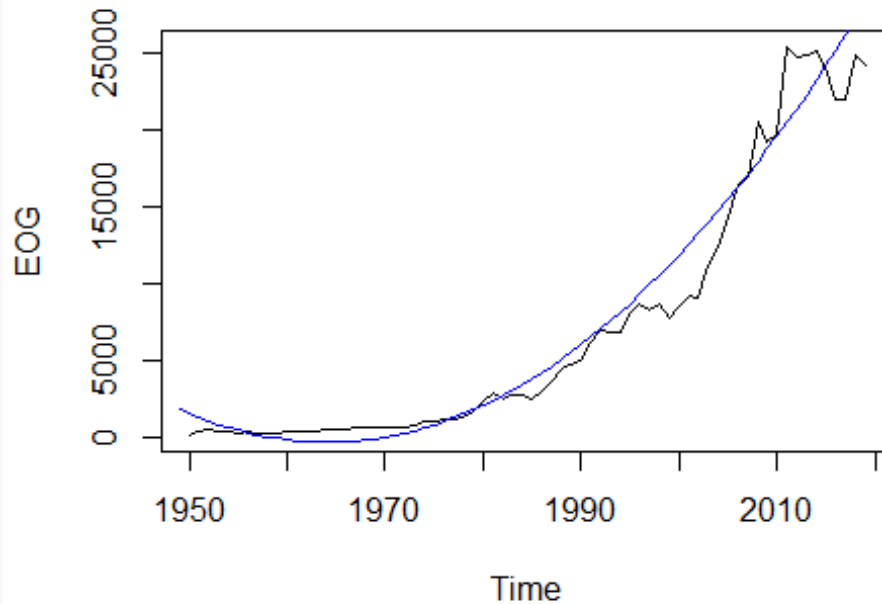
```
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

##

```
## Residual standard error: 4.819 on 67 degrees of freedom
```

```
## Multiple R-squared:  0.9814, Adjusted R-squared:  0.9809
## F-statistic: 1772 on 2 and 67 DF,  p-value: < 2.2e-16
```

All coefficients are reported as significant. Let us plot the time series against the fit.



We go on with the search for structural changes.

```
## Breakpoints at observation number:
```

```
##
## m = 1          48
## m = 2         45 59
## m = 3         35  53 63
## m = 4         23  47 56 63
## m = 5         23 33  48 56 63
## m = 6         23 33 40 48 56 63
## m = 7         7  23 33 40 48 56 63
## m = 8         7 16 23 33 40 48 56 63
```

```
##
## Corresponding to breakdates:
```

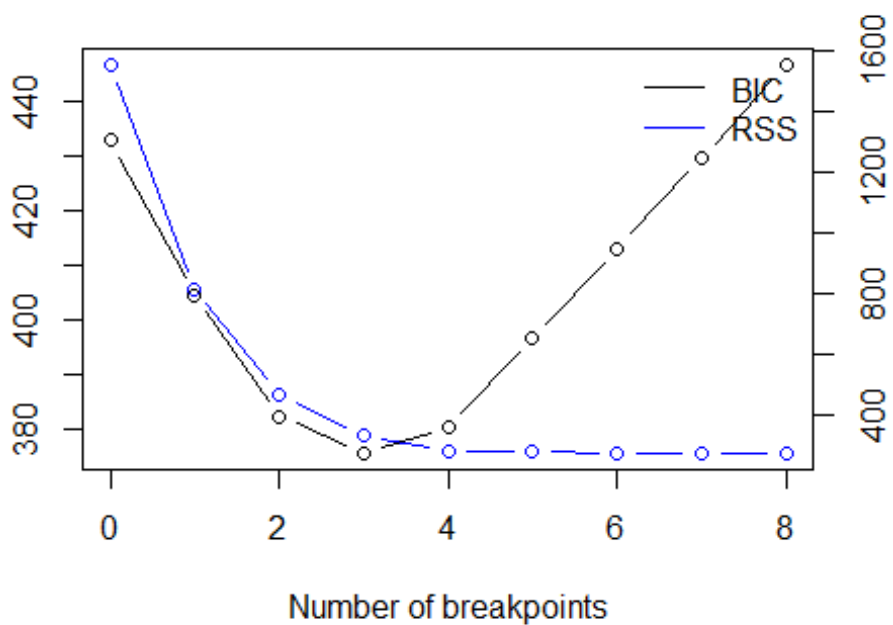
```
## m = 1          1997
## m = 2         1994 2008
## m = 3         1984          2002 2012
## m = 4         1972          1996 2005 2012
## m = 5         1972 1982          1997 2005 2012
## m = 6         1972 1982 1989 1997 2005 2012
```

```

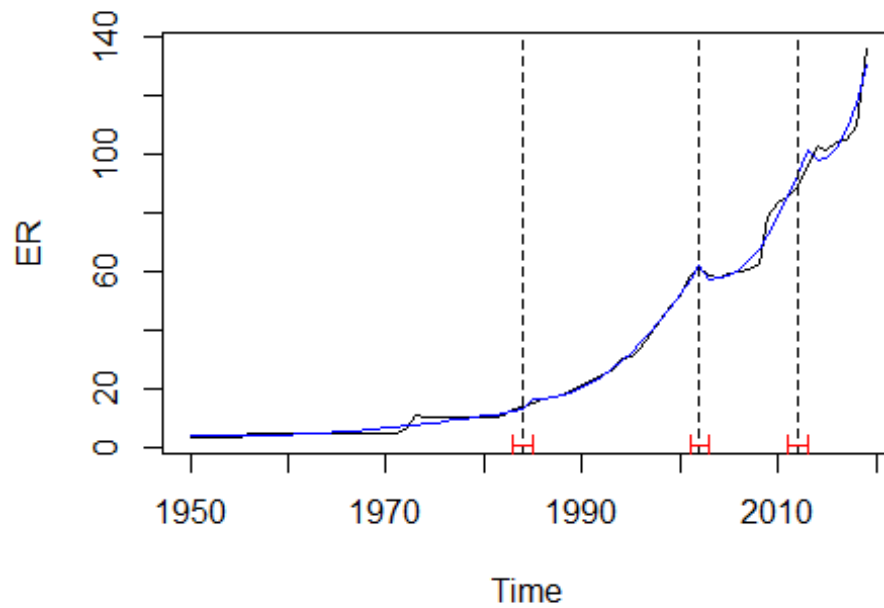
## m = 7  1956      1972 1982 1989 1997 2005 2012
## m = 8  1956 1965 1972 1982 1989 1997 2005 2012
##
## Fit:
##
## m  0      1      2      3      4      5      6      7      8
## RSS 1555.8  815.6  466.7  332.3  279.3  275.9  273.8  271.9  271.6
## BIC  432.7  404.5  382.4  375.7  380.5  396.6  413.1  429.6  446.5

```

BIC and Residual Sum of Squares



The BIC minimum value is reached for $m = 3$



Polynomial fit Break dates

1984 2002 2012

Polynomial fit Coefficients

##	(Intercept)	tt	I(tt^2)
## 1950 - 1984	3.735494	-0.06319485	0.00922888
## 1985 - 2002	217.052405	-11.21131047	0.15637110
## 2003 - 2012	1411.066513	-49.94630515	0.46067424
## 2013 - 2019	6680.616193	-201.22010357	1.53774643